Logico-Numerical Abstract Acceleration and Application to the Verification of Data-Flow Programs

Peter Schrammel and Bertrand Jeannet
{peter.schrammel,bertrand.jeannet}@inria.fr

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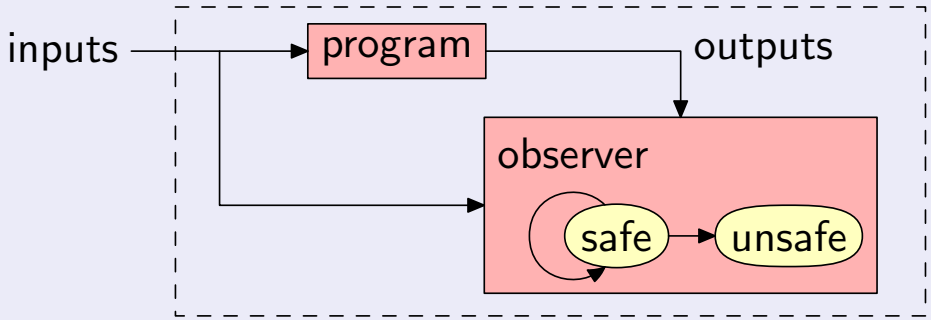
Motivation

Verification of safety properties about LUSTRE programs



- Check by reachability analysis whether "safe" is an invariant

Observations and goals

- Large Boolean state space
  - $\longrightarrow$ Avoid enumeration: use state space partitioning
- Mostly regular linear arithmetic
  - $\longrightarrow$ Exploit a specialized method: abstract acceleration

Reachability analysis of numerical programs

Abstract interpretation (Cousot & Cousot 1977)	over-approximation (e.g. intervals, octagons, polyhedra)	termination
Acceleration (Finkel & Leroux 2002)	exact (Presburger formula)	no guarantee for termination for non-flat systems

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What is acceleration?

- Method for analyzing loops in control flow graphs (CFG) of numerical programs
- Replace a loop transition τ by its reflexive and transitive closure τ^*

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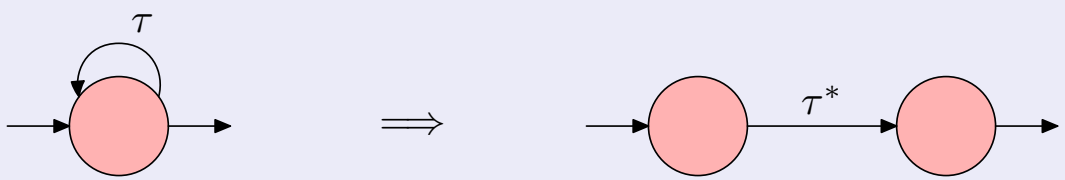

- Applicable to affine transitions $\tau : \mathbf{Ax} \leq \mathbf{b} \rightarrow \mathbf{x}' = \mathbf{Cx} + \mathbf{d}$
  - where $\{\mathbf{C}^p \mid p \geq 0\}$ is finite

Combination of acceleration and abstract interpretation

Abstract Acceleration (Gonnord & Halbwachs 2006)

- Computes a convex polyhedron $\tau^{\otimes}$ over-approximating the exact result τ^*

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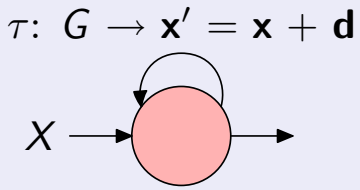

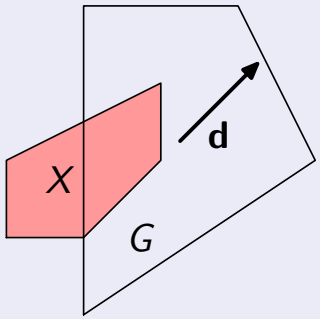

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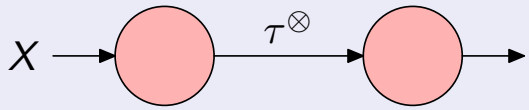

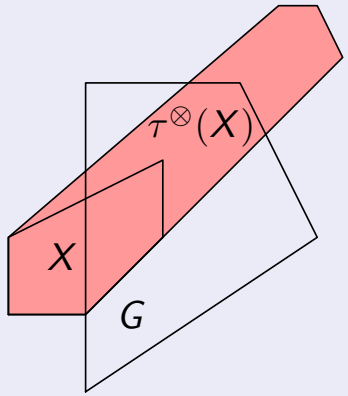

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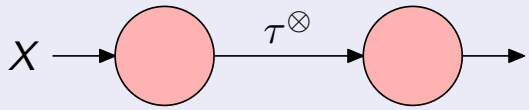

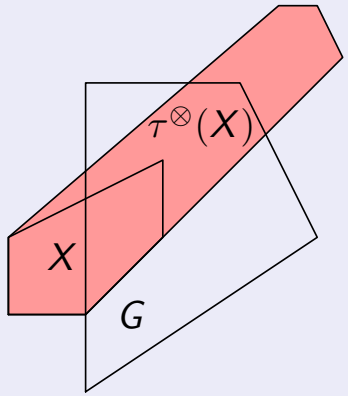

- Improves precision:
  - Computes $\tau^{\otimes} \approx (\alpha \circ \tau^* \circ \gamma)$ instead of $(\alpha \circ \tau \circ \gamma)^*$
  - No widening for flat systems
- Improves performance: fewer iterations

Application to LUSTRE programs?

Issues

- Numerical input variables $\surd$ (Schrammel & Jeannet 2010)
- Boolean state variables

Approaches

	abstract interpretation	
enumeration of Boolean states	*classical AI*	

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Our approach

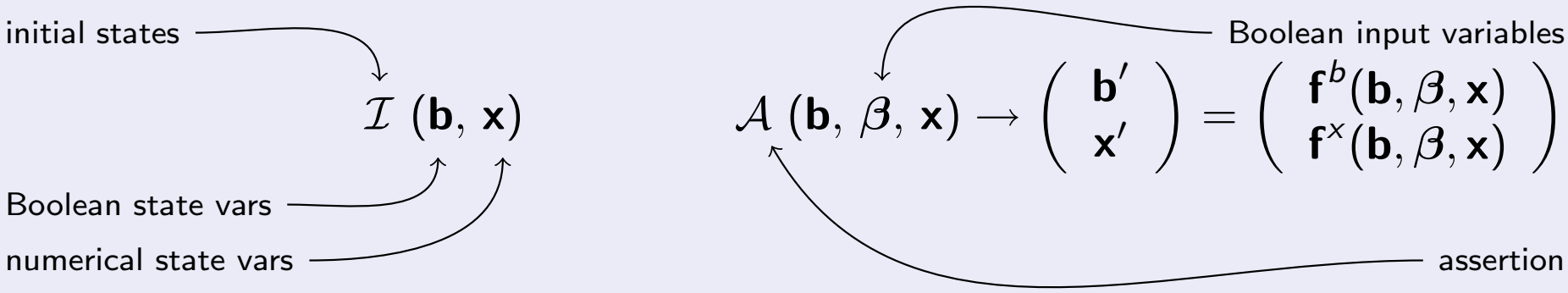
Using a logico-numerical abstract domain:

- Logico-numerical abstract acceleration method
- Partitioning heuristics

Outline

Outline

Program Model



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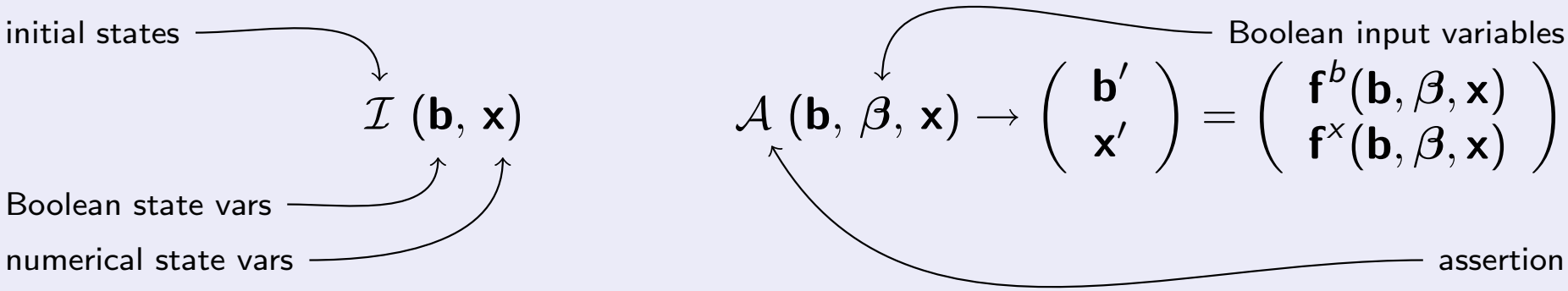


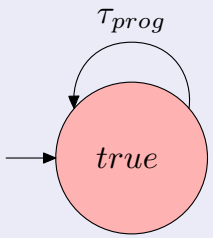
Example

$\mathcal{I}(\mathbf{b}, \mathbf{x}) = \neg b_0 \wedge \neg b_1 \wedge x_0 = 0 \wedge x_1 = 0 \wedge x_2 = 0$

$true \rightarrow$

$$\begin{cases} b_0' = b_0 \vee (\neg b_0 \wedge x_0 > 10 \wedge x_1 > 10) \\ b_1' = b_1 \vee (\neg b_1 \wedge x_0 > 20) \\ x_0' = \begin{cases} x_0 + 1 & \text{if } (\neg b_0 \wedge \neg b_1 \wedge x_0 \leq 10 \wedge \beta) \vee (b_0 \wedge \neg b_1 \wedge x_0 \leq 20) \\ 0 & \text{if } \neg b_0 \wedge \neg b_1 \wedge x_0 > 10 \wedge x_1 > 10 \\ x_0 & \text{otherwise} \end{cases} \\ x_1' = \begin{cases} x_1 + 1 & \text{if } \neg b_0 \wedge \neg b_1 \wedge x_1 \leq 10 \wedge \neg\beta \\ x_1 & \text{otherwise} \end{cases} \\ x_2' = \begin{cases} x_2 + 1 & \text{if } (\neg b_0 \wedge \neg b_1 \wedge (x_0 \leq 10 \wedge \beta \vee x_1 \leq 10 \wedge \neg\beta)) \vee (b_0 \wedge \neg b_1) \\ x_2 & \text{otherwise} \end{cases} \end{cases}$$

Classical Approach



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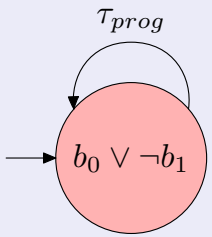

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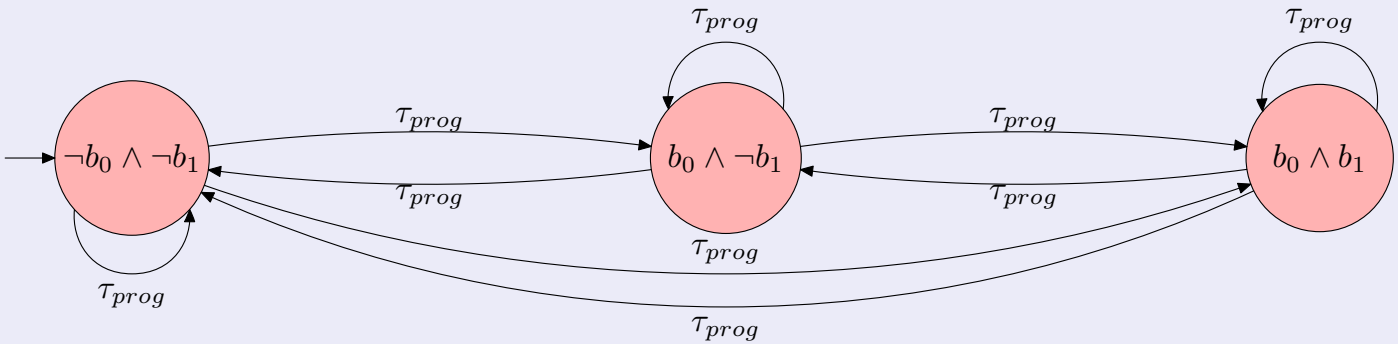
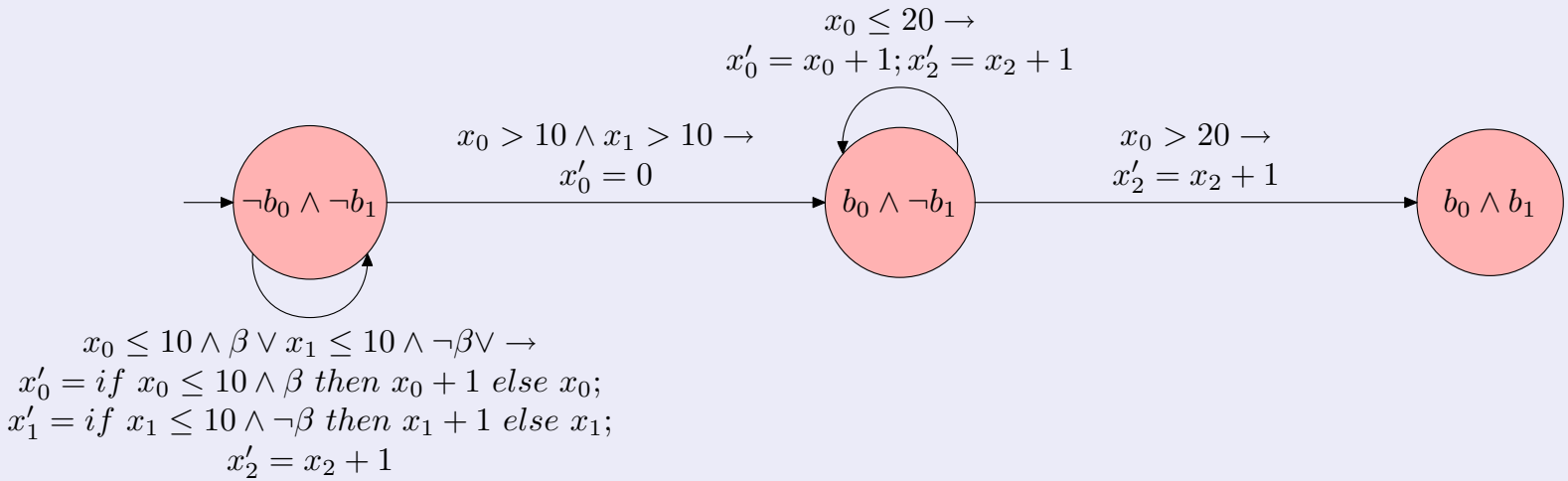


1. Boolean state space enumeration

Classical Approach



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2. Transition simplification by partial evaluation

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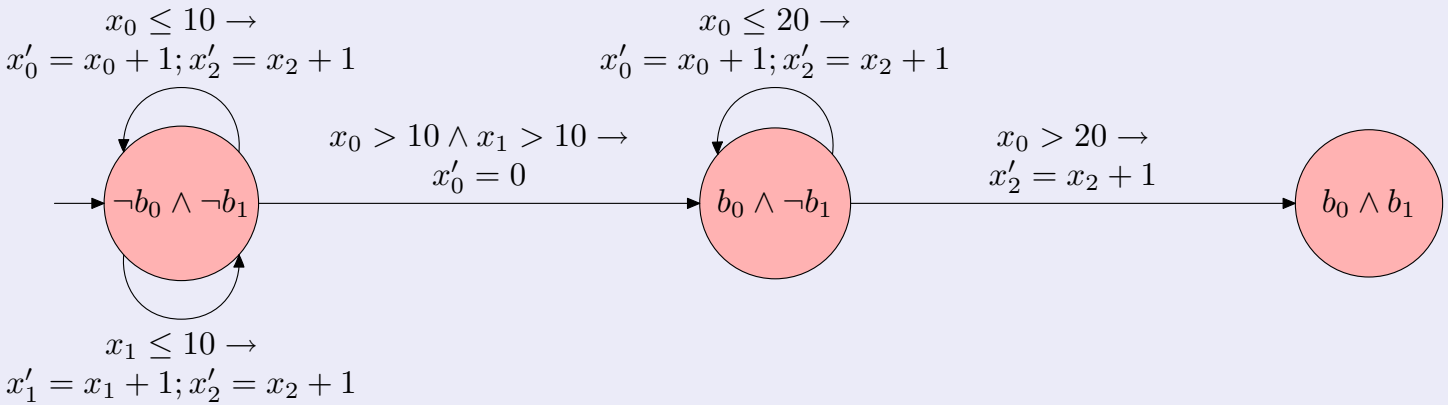

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2. Transition simplification by partial evaluation
3. Elimination of Boolean input variables

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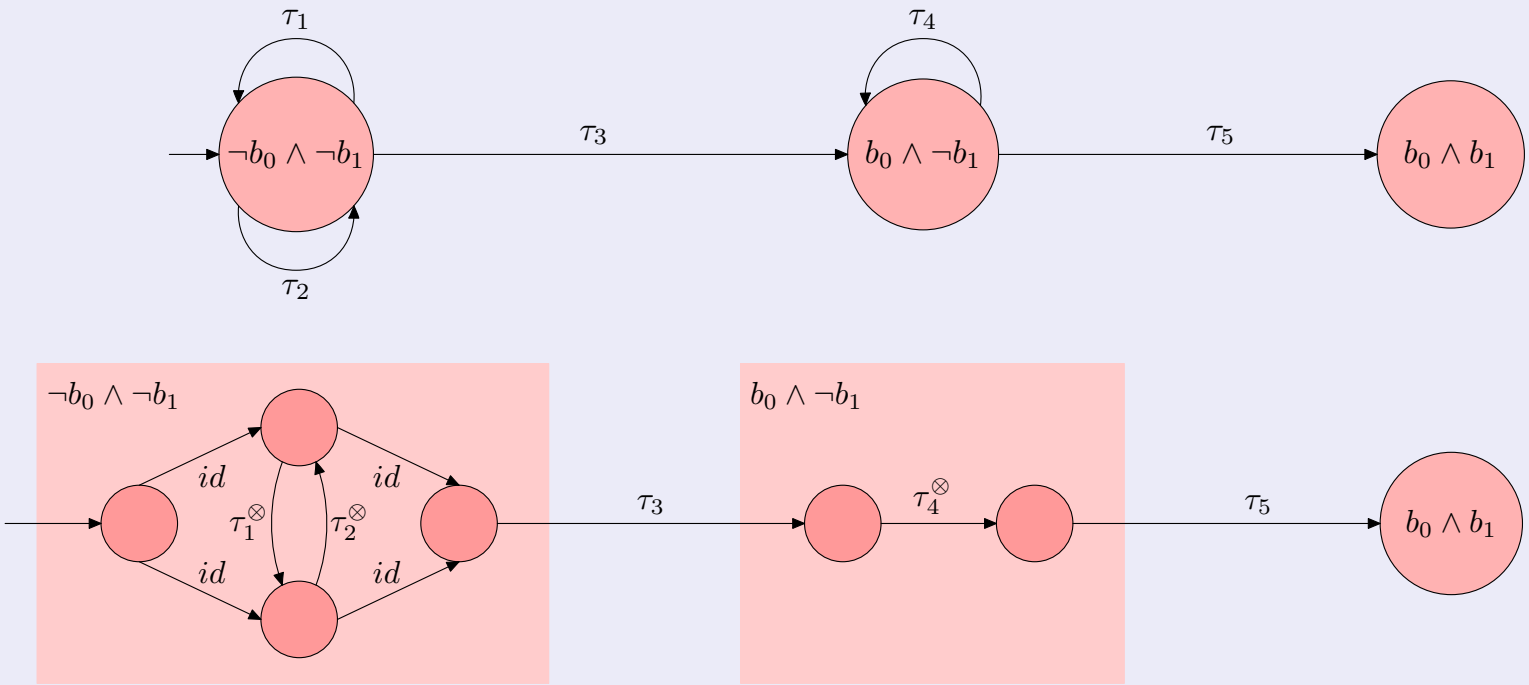

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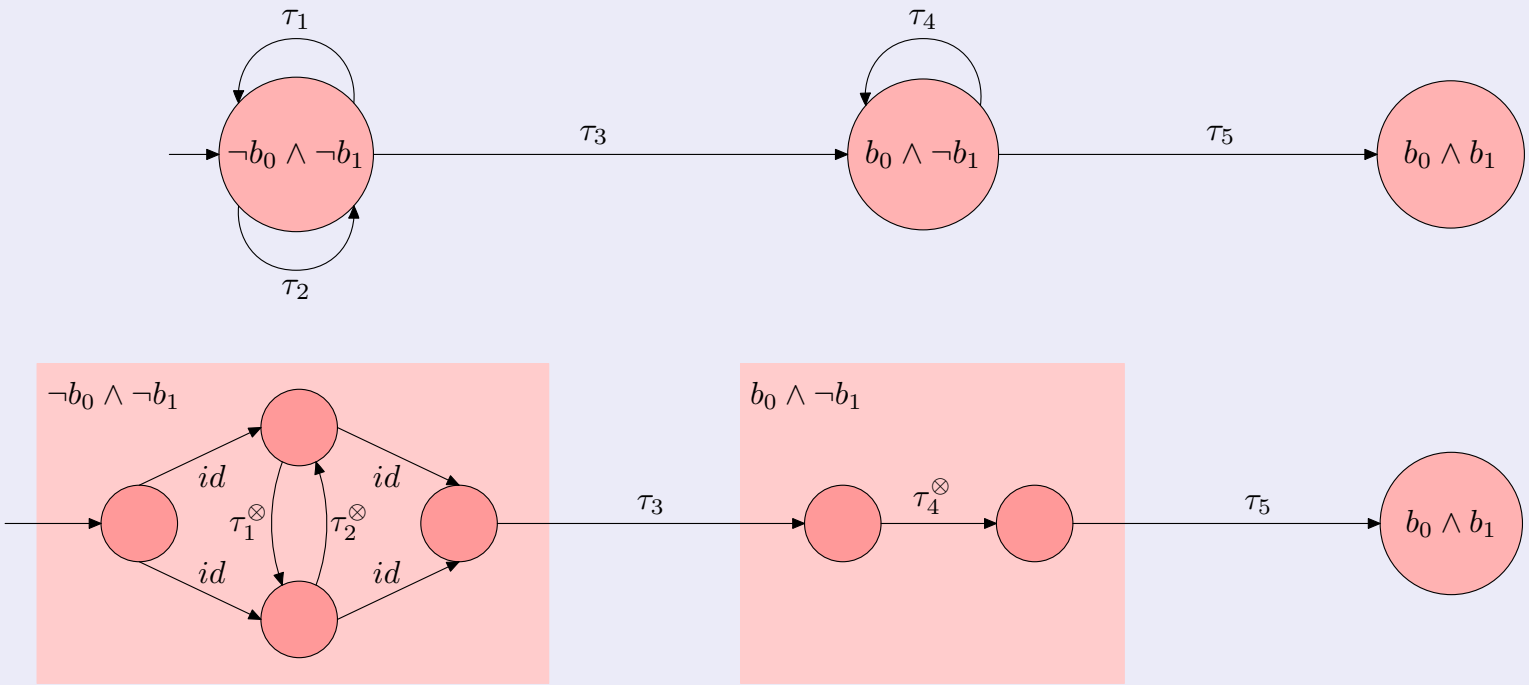

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5. Analysis

~~Classical~~ Our Approach

1. Boolean state space ~~enumeration~~ partitioning
2. Transition simplification by partial evaluation
3. Elimination of Boolean input variables
4. Expansion ("Flattening") of logico-numerical accelerable self-loops
5. Analysis

Outline

How to accelerate logico-numerical loops?

Numerically accelerable self-loop

$$\tau : g^b(\mathbf{b}) \wedge g^x(\mathbf{x}) \rightarrow \begin{pmatrix} \mathbf{b}' \\ \mathbf{x}' \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{a}(\mathbf{x}) \end{pmatrix}$$

Boolean part of transition function is identity

accelerable affine expression

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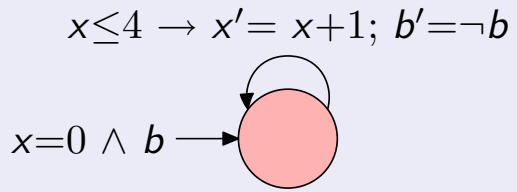
Problem:

- Accelerable self-loops might not exist at all
  - $\rightarrow$ Boolean identity too restrictive
  - $\rightarrow$ rarely applicable



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general Boolean expression

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Idea

Decoupling of numerical and Boolean parts of τ:

$$\tau_x : g^b(\mathbf{b}) \wedge g^x(\mathbf{x}) \rightarrow \begin{pmatrix} \mathbf{b}' \\ \mathbf{x}' \end{pmatrix} = \begin{pmatrix} \mathbf{b} \\ \mathbf{a}(\mathbf{x}) \end{pmatrix}$$

$$\tau_b : g^b(\mathbf{b}) \wedge g^x(\mathbf{x}) \rightarrow \begin{pmatrix} \mathbf{b}' \\ \mathbf{x}' \end{pmatrix} = \begin{pmatrix} f^b(\mathbf{b}, \mathbf{x}) \\ \mathbf{x} \end{pmatrix}$$

Boolean identity

numerical identity

Logico-Numerical Abstract Acceleration

Propositions

Decoupling abstraction: $\wp(\mathbb{B}^m \times \mathbb{R}^n) \underset{\pi}{\overset{id}{\leftrightarrows}} \wp(\mathbb{B}^m) \times \wp(\mathbb{R}^n)$

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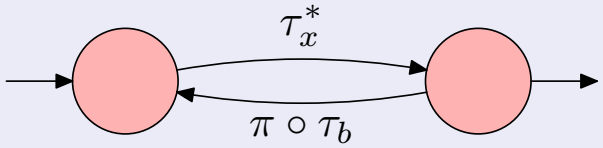

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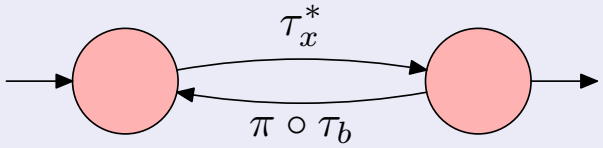

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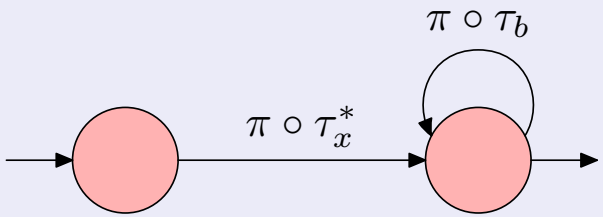

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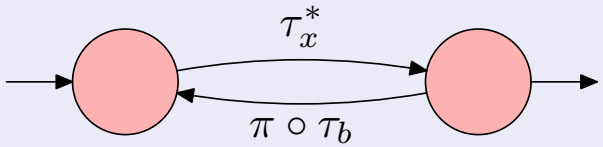

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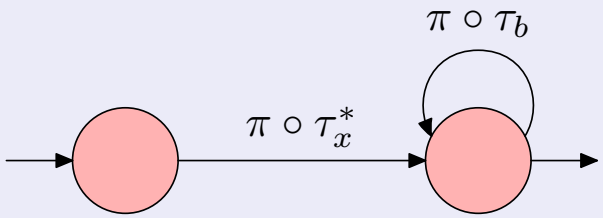

Logico-numerical accelerable self-loop

general Boolean expression

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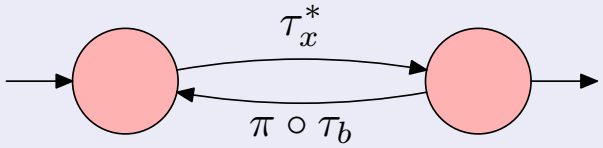

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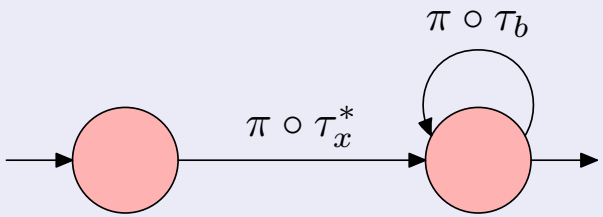

Theorem

With the abstraction (Jeannet 2003):

$$\begin{array}{rcl} \wp(\mathbb{B}^m \times \mathbb{R}^n) & \underset{\alpha}{\overset{id}{\leftrightarrows}} & \wp(\mathbb{B}^m) \times Pol(\mathbb{R}^n) \\ S(\mathbf{b}, \mathbf{x}) & \rightarrow & \varphi(\mathbf{b}) \wedge Pol(\mathbf{x}) \end{array}$$

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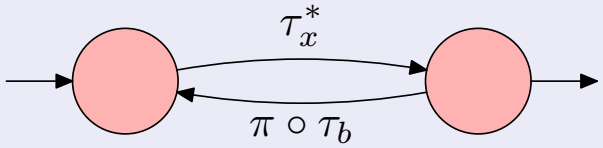

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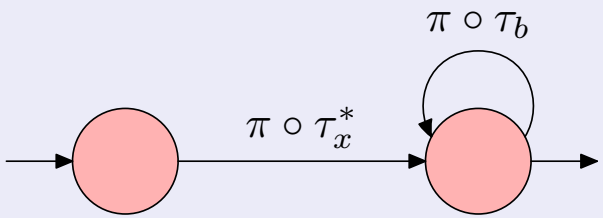

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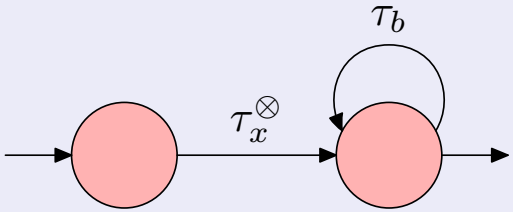
$\tau^* \subseteq \tau^{\otimes}$



Discussion

Advantages

- Accelerate self-loops while fighting state-space explosion using partitioning

Drawback

- Decoupling may lose precision

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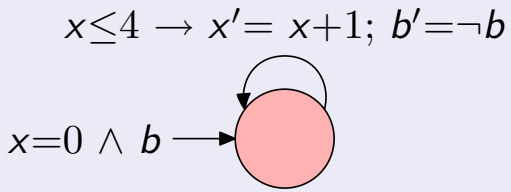
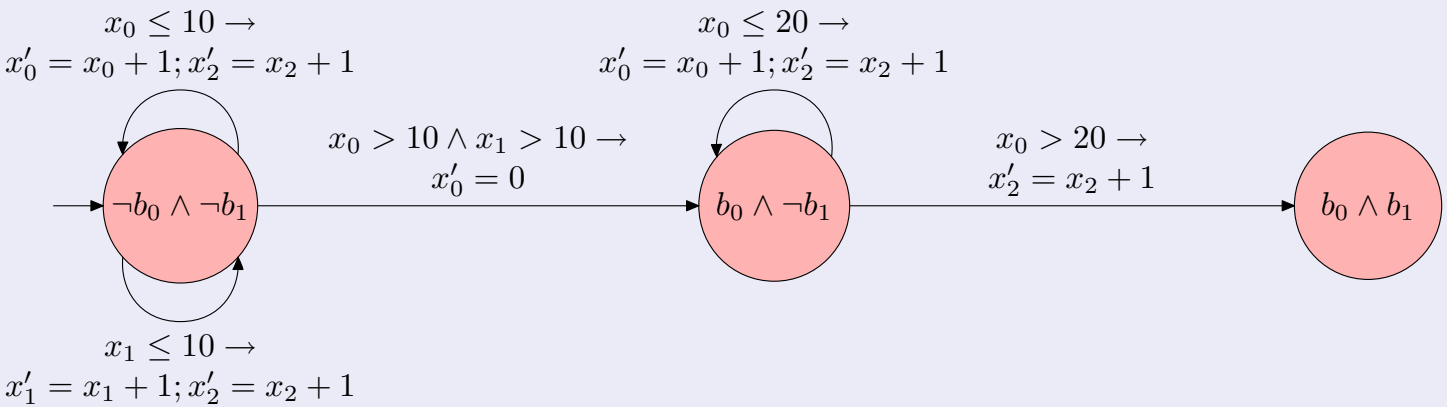
Observations

- Partial decoupling only
- Loses mostly that information that is not representable in the abstract domain anyway
  - Exact reachable set: $\{\mathit{true}\} \times \{0, 2, 4\} \cup \{\mathit{false}\} \times \{1, 3, 5\}$
  - In the abstract domain: $\{\top\} \times \{0 \leq x \leq 5\}$



Discussion

Example: Precision



Standard analysis on enumerated CFG

$$0 \leq x_0 \quad \wedge \quad 0 \leq x_1 \quad \wedge \quad x_0 + x_1 \leq x_2$$

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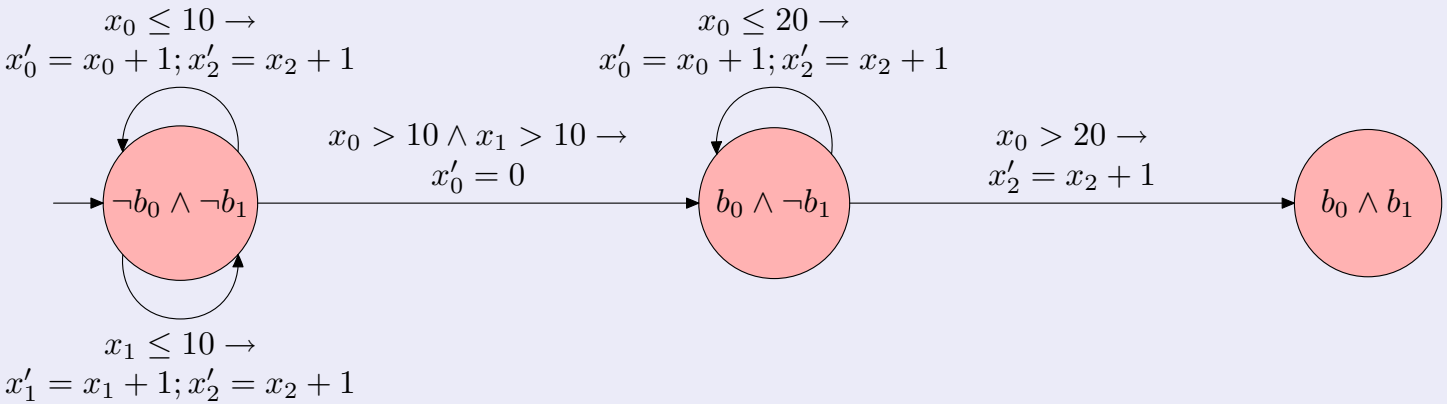
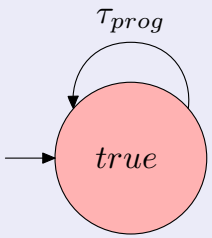


Standard analysis on enumerated CFG	$0 \leq x_0$	$\wedge$	$0 \leq x_1$	$\wedge$	$x_0 + x_1 \leq x_2$
Abstract acceleration on enumerated CFG	$0 \leq x_0 \leq 21$	$\wedge$	$0 \leq x_1 \leq 11$	$\wedge$	$x_0 + x_1 \leq x_2 \leq 44$

Discussion

Example: Precision



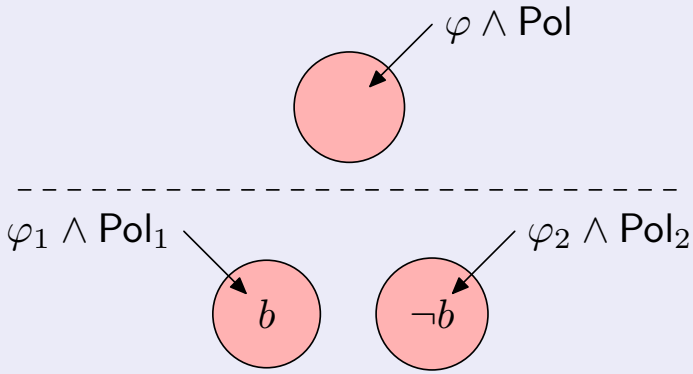
Standard analysis on enumerated CFG	$0 \leq x_0$	$\wedge$	$0 \leq x_1$	$\wedge$	$x_0 + x_1 \leq x_2$
Abstract acceleration on enumerated CFG	$0 \leq x_0 \leq 21$	$\wedge$	$0 \leq x_1 \leq 11$	$\wedge$	$x_0 + x_1 \leq x_2 \leq 44$
Logico-numerical abstract accel. on single location	$0 \leq x_0 \leq 21$	$\wedge$	$0 \leq x_1 \leq 11$	$\wedge$	$x_0 + x_1 \leq x_2$

Outline

Partitioning

Why partitioning?

- Trade-off between performance and precision:
  - Reasonably small CFG to escape state-space explosion
  - Sufficiently large CFG to benefit from:
    - ⋆ $\longrightarrow$ Disjunctive abstract domain
    - ⋆ $\longrightarrow$ Less precision loss in convex unions
    - ⋆ $\longrightarrow$ More targeted application of *widening*



- Limit approximations made by our logico-numerical acceleration method

Partitioning

Our heuristics

- Boolean states with same numerical behavior grouped in same location:
  - $\longrightarrow$ Numerical equations independent of Boolean equations
  - $\longrightarrow$ Reduced impact of decoupling

$$\mathbf{b}_1 \sim \mathbf{b}_2 \Leftrightarrow \begin{cases} \forall \boldsymbol{\beta}_1, \mathcal{C} : & \mathcal{A}(\mathbf{b}_1, \boldsymbol{\beta}_1, \mathcal{C}) \Rightarrow \\ & \exists \boldsymbol{\beta}_2 : \mathcal{A}(\mathbf{b}_2, \boldsymbol{\beta}_2, \mathcal{C}) \wedge f^x(\mathbf{b}_1, \boldsymbol{\beta}_1, \mathcal{C}) = f^x(\mathbf{b}_2, \boldsymbol{\beta}_2, \mathcal{C}) \\ \textit{and} & \textit{vice versa} \end{cases}$$

Partitioning

Our heuristics

- Boolean states with same numerical behavior grouped in same location:
  - $\longrightarrow$ Numerical equations independent of Boolean equations
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$$\mathbf{b}_1 \sim \mathbf{b}_2 \Leftrightarrow \left\{ \begin{array}{ll} \forall \boldsymbol{\beta}_1, \mathcal{C} : & \mathcal{A}(\mathbf{b}_1, \boldsymbol{\beta}_1, \mathcal{C}) \Rightarrow \\ & \exists \boldsymbol{\beta}_2 : \mathcal{A}(\mathbf{b}_2, \boldsymbol{\beta}_2, \mathcal{C}) \wedge f^x(\mathbf{b}_1, \boldsymbol{\beta}_1, \mathcal{C}) = f^x(\mathbf{b}_2, \boldsymbol{\beta}_2, \mathcal{C}) \\ \textit{and} & \textit{vice versa} \end{array} \right.$$

Example

$$x' = \left\{ \begin{array}{ll} x+1 & \text{if } b_1 \wedge \beta_1 \wedge x \leq 10 \\ x+1 & \text{if } b_2 \wedge \beta_2 \wedge x \leq 10 \\ x-2 & \text{if } \neg b_1 \wedge \neg b_2 \wedge x \geq 0 \\ x & \text{otherwise} \end{array} \right.$$

Partitioning

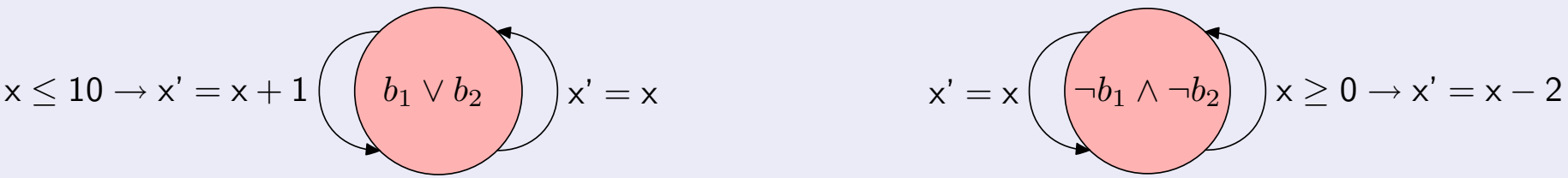
Our heuristics

- Boolean states with same numerical behavior grouped in same location:
  - $\rightarrow$ Numerical equations independent of Boolean equations
  - $\rightarrow$ Reduced impact of decoupling

$$\mathbf{b}_1 \sim \mathbf{b}_2 \Leftrightarrow \begin{cases} \forall \beta_1, \mathcal{C} : & \mathcal{A}(\mathbf{b}_1, \beta_1, \mathcal{C}) \Rightarrow \\ & \exists \beta_2 : \mathcal{A}(\mathbf{b}_2, \beta_2, \mathcal{C}) \wedge f^x(\mathbf{b}_1, \beta_1, \mathcal{C}) = f^x(\mathbf{b}_2, \beta_2, \mathcal{C}) \\ \textit{and} & \textit{vice versa} \end{cases}$$

Example

$$x' = \begin{cases} x+1 & \text{if } b_1 \wedge \beta_1 \wedge x \leq 10 \\ x+1 & \text{if } b_2 \wedge \beta_2 \wedge x \leq 10 \\ x-2 & \text{if } \neg b_1 \wedge \neg b_2 \wedge x \geq 0 \\ x & \text{otherwise} \end{cases}$$



Outline

Experimental Evaluation

Tool NBACCEL

- Uses the abstract domain library BDDAPRON

Comparison

	abstract interpretation	abstract acceleration
enumeration of Boolean states	*classical AI*	ASPIC (Gonnord 2009)
state space partitioning	NBAC (Jeannet 2003) – property-guided incremental partitioning of Boolean and numerical states	NBACCEL (this talk)

Experimental Evaluation

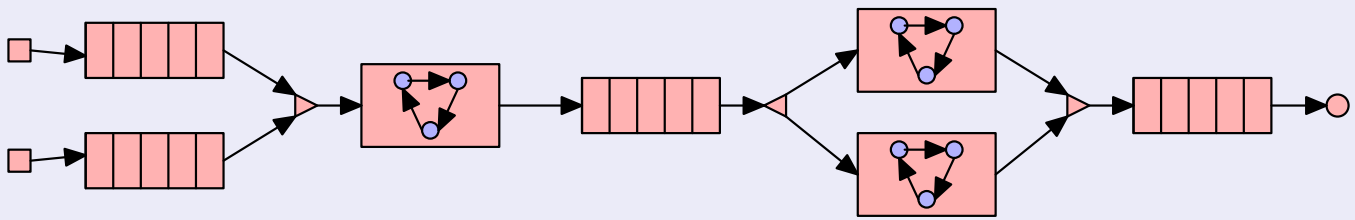
Tool NBACCEL

- Uses the abstract domain library BDDAPRON

	abstract interpretation	abstract acceleration
enumeration of Boolean states	*classical AI*	ASPIC (Gonnord 2009)
state space partitioning	NBAC (Jeannet 2003)	NBACCEL (this talk)

Benchmarks

- Models of production lines: sources, sinks, buffers, machines, etc
- Communication via handshakes: large Boolean state space
- All numerical actions are accelerable
- Properties that depend on numerical variables: e.g. throughput time



Experimental Evaluation

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
Gate 1	4/4/2	7	?	5	**0.73**	24	?
Escalator 1	5/4/2	12	**0.14 (0.04)**	9	0.49	22	?
Traffic 1	4/6/0	18	**0.14 (0.01)**	16	0.19	5	3.49
Traffic 2	4/8/0	18	?	16	**0.35**	28	?
LCM Quest 0a-1	7/2/0	7	**0.04 (0.01)**	5	**0.04**	5	0.05
LCM Quest 0a-2	7/3/0	6	**0.05 (0.01)**	4	**0.05**	8	0.19
LCM Quest 0b-1	10/3/0	19	**0.08 (0.01)**	12	**0.08**	9	?
LCM Quest 0b-2	10/4/0	17	**0.09 (0.01)**	11	0.20	33	?
LCM Quest 0c-1	15/4/0	28	0.17 (0.01)	16	**0.16**	8	0.86
LCM Quest 0c-2	15/5/0	25	**0.20 (0.05)**	14	0.24	50	14.8
LCM Quest 1b-1	16/5/0	55	0.92 (0.04)	29	**0.37**	15	?
LCM Quest 1b-2	16/5/0	45	0.76 (0.12)	23	**0.47**	61	?
LCM Quest 1-1	16/5/0	114	1.99 (0.48)	42	**0.92**	6	2.45
LCM Quest 1-2	16/6/0	100	?	34	?	>156	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation

locations in enumerated CFG

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
Gate 1	4/4/2	7	?	5	**0.73**	24	?
Escalator 1	5/4/2	12	**0.14 (0.04)**	9	0.49	22	?
Traffic 1	4/6/0	18	**0.14 (0.01)**	16	0.19	5	3.49
Traffic 2	4/8/0	18	?	16	**0.35**	28	?
LCM Quest 0a-1	7/2/0	7	**0.04 (0.01)**	5	**0.04**	5	0.05
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LCM Quest 0b-1	10/3/0	19	**0.08 (0.01)**	12	**0.08**	9	?
LCM Quest 0b-2	10/4/0	17	**0.09 (0.01)**	11	0.20	33	?
LCM Quest 0c-1	15/4/0	28	0.17 (0.01)	16	**0.16**	8	0.86
LCM Quest 0c-2	15/5/0	25	**0.20 (0.05)**	14	0.24	50	14.8
LCM Quest 1b-1	16/5/0	55	0.92 (0.04)	29	**0.37**	15	?
LCM Quest 1b-2	16/5/0	45	0.76 (0.12)	23	**0.47**	61	?
LCM Quest 1-1	16/5/0	114	1.99 (0.48)	42	**0.92**	6	2.45
LCM Quest 1-2	16/6/0	100	?	34	?	>156	>

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Experimental Evaluation

locations in enumerated CFG

total

analysis only

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/x/β	size	time	size	time	size	time
Gate 1	4/4/2	7	?	5	**0.73**	24	?
Escalator 1	5/4/2	12	**0.14 (0.04)**	9	0.49	22	?
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LCM Quest 0a-1	7/2/0	7	**0.04 (0.01)**	5	**0.04**	5	0.05
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LCM Quest 1b-2	16/5/0	45	0.76 (0.12)	23	**0.47**	61	?
LCM Quest 1-1	16/5/0	114	1.99 (0.48)	42	**0.92**	6	2.45
LCM Quest 1-2	16/6/0	100	?	34	?	>156	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation

locations in enumerated CFG

total

analysis only

no abstract acceleration

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
Gate 1	4/4/2	7	?	5	**0.73**	24	?
Escalator 1	5/4/2	12	**0.14 (0.04)**	9	0.49	22	?
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Traffic 2	4/8/0	18	?	16	**0.35**	28	?
LCM Quest 0a-1	7/2/0	7	**0.04 (0.01)**	5	**0.04**	5	0.05
LCM Quest 0a-2	7/3/0	6	**0.05 (0.01)**	4	**0.05**	8	0.19
LCM Quest 0b-1	10/3/0	19	**0.08 (0.01)**	12	**0.08**	9	?
LCM Quest 0b-2	10/4/0	17	**0.09 (0.01)**	11	0.20	33	?
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LCM Quest 1-2	16/6/0	100	?	34	?	>156	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation (cont)

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
LCM Quest 2-1	17/6/0	247	c	82	**7.84**	9	12.8
LCM Quest 2-2	17/7/0	198	>	62	?	>76	>
LCM Quest 3d-1	26/6/0	281	>	81	**5.43**	49	?
LCM Quest 3d-2	26/7/0	266	c	73	?	446	?
LCM Quest 3-1	25/5/0	483	26.5 (14.4)	58	8.49	12	**3.76**
LCM Quest 3-2	25/6/0	481	c	54	?	>1173	>
LCM Quest 3e-1	27/7/0	638	>	140	**20.6**	49	?
LCM Quest 3e-2	27/8/0	514	>	110	**6.46**	>28	>
LCM Quest 3c-1	26/6/0	1319	>	130	**34.2**	9	?
LCM Quest 3c-2	26/7/0	1056	c	98	?	>70	>
LCM Quest 3b-1	26/6/0	1724	>	170	43.8	14	**19.1**
LCM Quest 3b-2	26/7/0	1710	>	162	?	>32	>
LCM Quest 4-1	27/7/0	4482	>	386	186	9	**50.1**
LCM Quest 4-2	27/8/0	3586	>	290	?	>6	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation (cont)

state space explosion

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/x/β	size	time	size	time	size	time
LCM Quest 2-1	17/6/0	247	c	82	**7.84**	9	12.8
LCM Quest 2-2	17/7/0	198	>	62	?	>76	>
LCM Quest 3d-1	26/6/0	281	>	81	**5.43**	49	?
LCM Quest 3d-2	26/7/0	266	c	73	?	446	?
LCM Quest 3-1	25/5/0	483	26.5 (14.4)	58	8.49	12	**3.76**
LCM Quest 3-2	25/6/0	481	c	54	?	>1173	>
LCM Quest 3e-1	27/7/0	638	>	140	**20.6**	49	?
LCM Quest 3e-2	27/8/0	514	>	110	**6.46**	>28	>
LCM Quest 3c-1	26/6/0	1319	>	130	**34.2**	9	?
LCM Quest 3c-2	26/7/0	1056	c	98	?	>70	>
LCM Quest 3b-1	26/6/0	1724	>	170	43.8	14	**19.1**
LCM Quest 3b-2	26/7/0	1710	>	162	?	>32	>
LCM Quest 4-1	27/7/0	4482	>	386	186	9	**50.1**
LCM Quest 4-2	27/8/0	3586	>	290	?	>6	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation (cont)

state space explosion

CFG 10 times smaller

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
LCM Quest 2-1	17/6/0	247	c	82	**7.84**	9	12.8
LCM Quest 2-2	17/7/0	198	>	62	?	>76	>
LCM Quest 3d-1	26/6/0	281	>	81	**5.43**	49	?
LCM Quest 3d-2	26/7/0	266	c	73	?	446	?
LCM Quest 3-1	25/5/0	483	26.5 (14.4)	58	8.49	12	**3.76**
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LCM Quest 3e-1	27/7/0	638	>	140	**20.6**	49	?
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LCM Quest 3c-1	26/6/0	1319	>	130	**34.2**	9	?
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LCM Quest 3b-1	26/6/0	1724	>	170	43.8	14	**19.1**
LCM Quest 3b-2	26/7/0	1710	>	162	?	>32	>
LCM Quest 4-1	27/7/0	4482	>	386	186	9	**50.1**
LCM Quest 4-2	27/8/0	3586	>	290	?	>6	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Experimental Evaluation (cont)

state space explosion

CFG 10 times smaller

incremental partitioning

	vars	ASPIC (Gonnord 2009)		NBACCEL (this talk)		NBAC (Jeannet 2003)	
	b/**x**/β	size	time	size	time	size	time
LCM Quest 2-1	17/6/0	247	c	82	**7.84**	9	12.8
LCM Quest 2-2	17/7/0	198	>	62	?	>76	>
LCM Quest 3d-1	26/6/0	281	>	81	**5.43**	49	?
LCM Quest 3d-2	26/7/0	266	c	73	?	446	?
LCM Quest 3-1	25/5/0	483	26.5 (14.4)	58	8.49	12	**3.76**
LCM Quest 3-2	25/6/0	481	c	54	?	>1173	>
LCM Quest 3e-1	27/7/0	638	>	140	**20.6**	49	?
LCM Quest 3e-2	27/8/0	514	>	110	**6.46**	>28	>
LCM Quest 3c-1	26/6/0	1319	>	130	**34.2**	9	?
LCM Quest 3c-2	26/7/0	1056	c	98	?	>70	>
LCM Quest 3b-1	26/6/0	1724	>	170	43.8	14	**19.1**
LCM Quest 3b-2	26/7/0	1710	>	162	?	>32	>
LCM Quest 4-1	27/7/0	4482	>	386	186	9	**50.1**
LCM Quest 4-2	27/8/0	3586	>	290	?	>6	>

?	"don´t know" (property not proved)
>	timed out after 600s
c	out of memory or crashed

Outline

Alternative Methods for Logico-Numerical Programs

- Bultan et al 2000:
  - Model-checking of logico-numerical programs using BDDs and Presburger formulae
  - Neither acceleraton nor widening
- Fränzle and Herde 2007 (HySat):
  - Bounded model-checking for hybrid systems using constraint solving
  - Boolean control structure encoded in linear pseudo-Boolean constraints
- Hagen and Tinelli 2008 (Kind):
  - k-induction-based verification of data-flow programs using SMT-solvers

Conclusion

- Logico-numerical abstract acceleration method
  - based on partial decoupling of Boolean and numerical transitions
- Dedicated partitioning technique
  - that makes logico-numerical abstract acceleration effective

	abstract interpretation	abstract acceleration
enumeration of Boolean states	*classical AI*	ASPIC
state space partitioning	NBAC	NBACCEL

Results:

- Improved precision and performance
- Properties proved on much smaller CFGs

Future Work:

- Incremental partition refinement
- Partitioning by numerical constraints

Logico-Numerical Abstract Acceleration and Application to the Verification of Data-Flow Programs

Peter Schrammel and Bertrand Jeannet
{peter.schrammel,bertrand.jeannet}@inria.fr

INRIA Grenoble – Rhône-Alpes

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