The Complexity of Abduction for Separated Heap Abstractions

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- Further papers extend the analysis, apply it to other domains.
- The published algorithms for abduction are incomplete.
- Is there a complete algorithm? (is the problem decidable?).
- If yes, what is the complexity for a common abstract domain?

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- Reachability?

$$\left\{ \begin{array}{c} \forall z.\mathsf{reach}(x,z) \Rightarrow \neg\mathsf{reach}(y,z) \wedge \\ \forall w.\mathsf{reach}(y,w) \Rightarrow \neg\mathsf{reach}(x,w) \wedge \\ \mathsf{ls}(x,0) \wedge \mathsf{ls}(y,0) \end{array} \right\}$$

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- Separation Logic?

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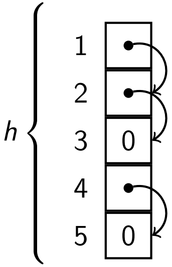

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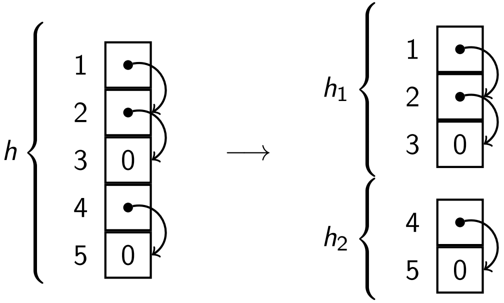

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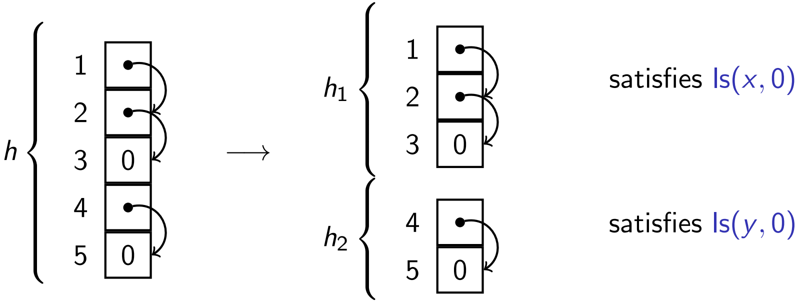

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  I.e., non-empty, acyclic list segments.

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$$x \neq y \wedge w \neq z \wedge x \mapsto y * \mathsf{ls}(y, x)$$

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- Cases occuring in practice can be usually treated in polytime.
- There is a polytime algorithm for a fixed number of lists.