

Abstract Domains of Affine Relations

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Efficiently find sound linear equalities
in programs using machine integers

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Machine integers = w -bit ints

Analyzing ints

Difficulties:

- Arithmetic overflow: $12 + 7 \equiv_{16} 3$
- All even numbers are zero divisors: $2 \cdot 8 \equiv_{16} 0$
- All odd numbers have inverses: $3 \cdot 11 \equiv_{16} 1$

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Advantages:

- Can represent some bit-level properties in linear equations:
 $8x \equiv_{16} 8$ means x is odd
- Fast operations on native ints
- Int domains are finite lattices
- Soundness: capture real semantics

KS: Conjunction of affine constraints

MOS: Affine set of affine transformers

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KS and MOS are **two-vocabulary** domains

Two-vocabulary domain

Definition

If the set of concrete program states is S ,
a **two-vocabulary domain** abstracts relations from S to S

“Standard” domains abstract program states,
Two-vocabulary domains abstract program transitions

Also called:

- Sharir-Pneuli-style domain
- Transformer domain
- Transition domain

Question 1

How can we adapt KS to directly model w -bit ints?

Symbolic Functions

Definition

Symbolic abstraction converts logical formulas to overapproximating domain elements

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Symbolic concretization converts domain elements to overapproximating logical formulas

Question 2

How can we perform
symbolic abstraction to MOS?

Question 3

KS and MOS:

Which is more precise?

Which is more efficient?

- How can we adapt KS to directly model w -bit ints?
- How can we perform symbolic abstraction to MOS?
- Which is more precise, KS or MOS?
- Which is more efficient, KS or MOS?

- How can we adapt KS to directly model w -bit ints?
- ~~How can we perform symbolic abstraction to MOS?~~
- Which is more precise, KS or MOS?
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How can we adapt KS to directly model w -bit ints?

KS Definition

KS element: Matrix of w -bit ints; each row encodes a constraint

Example

$$\begin{array}{ccccc} x & y & x' & y' & 1 \\ \left[\begin{array}{ccccc} 5 & 7 & 9 & 12 & 6 \\ 5 & 1 & 9 & 2 & 8 \end{array} \right] & : & & & \end{array} \quad \begin{array}{l} 5x + 7y + 9x' + 12y' + 6 = 0 \\ \text{and } 5x + y + 9x' + 2y' + 8 = 0 \end{array}$$

Generalization of: King and Søndergaard, CAV 2008

KS Definition

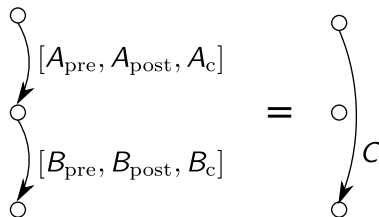
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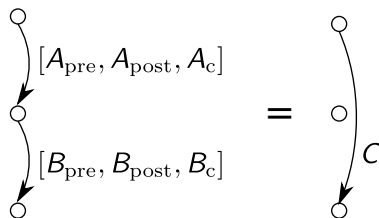
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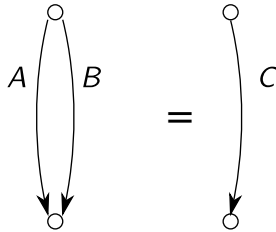
KS Compose

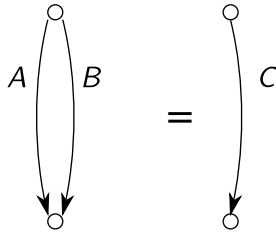


KS Compose



$$C = \text{Project} \left(\begin{bmatrix} A_{\text{pre}} & A_{\text{post}} & 0 & A_c \\ 0 & B_{\text{pre}} & B_{\text{post}} & B_c \end{bmatrix} \right)$$





$$C = \text{Project} \left(\begin{pmatrix} -A & A \\ B & 0 \end{pmatrix} \right)$$

Project is the critical operation!

Naive Project

- 1 Move lost variables to the left
- 2 Do Gaussian elimination
- 3 Drop every row constraining a lost variable
- 4 Drop the lost-variable columns

Example (Project onto x and x')

$$\begin{array}{ccccc} x & y & x' & y' & 1 \\ \left[\begin{array}{ccccc} 9 & 5 & 12 & 7 & 6 \\ 9 & 5 & 2 & 3 & 8 \end{array} \right] \end{array}$$

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Definition

The **row space** of a matrix is the set of linear combinations of its rows

Definition

The **null space** of a matrix is the set of values whose product with the matrix is zero

The null space of A is $\{x \mid Ax = 0\}$

Definition

A **row operation** is a matrix transformation that adds or changes individual rows without changing the matrix's row space

Some Row Operations:

- Scale a row by an **odd** number
- Add a multiple of one row to another
- Insert some multiple of a row into the matrix

Row Operations

What if we scale a row by an even number?

Example

Scale first row by 2:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Row Operations

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First row space: $[x \ y \ 0]$ for any x , any y

Second row space: $[x \ y \ 0]$ for **even** x , any y

Row-Echelon form

Definition

A matrix is in **Row-Echelon Form** if each row has fewer leading zeroes than the next row

Example

$$\begin{bmatrix} * & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * \end{bmatrix}$$

Howell form is a normal form for int matrices

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Like Gaussian Elimination,
Howellization preserves the row space

Properties of Howell Form

- Normal form for row spaces
- Normal form for null spaces
- Normal form for KS

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- Normal form for row spaces
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Simplifies checking KS equality

Howell Form Definition

The matrix is in Row-Echelon form

Example

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Howell Form Definition

Leading values are powers of 2

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Howell Form Definition

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Example

$$\begin{bmatrix} 1 & 11 & 5 & 12 & 14 \\ 0 & 4 & 0 & 2 & 6 \end{bmatrix}$$

Howell Form Definition

Leading values are largest in their columns

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$$\begin{bmatrix} 1 & 11 & 5 & 12 & 14 \\ 0 & 4 & 0 & 2 & 6 \end{bmatrix}$$

Howell Form Definition

Leading values are largest in their columns

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$$\begin{bmatrix} 1 & 3 & 5 & 8 & 2 \\ 0 & 4 & 0 & 2 & 6 \end{bmatrix}$$

Howell Form Definition

Every consequence of every row
is a linear combination of the matrix rows
that have at least as many leading zeros as the consequence

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Definition

The vectors $2^k v$ are the **consequences** of v

Example

The consequences of $[0 \ 4 \ 0 \ 2 \ 6]$ are
 $\{ [0 \ 8 \ 0 \ 4 \ 4], [0 \ 0 \ 0 \ 8 \ 8], [0 \ 0 \ 0 \ 0 \ 0] \}$

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Example

$$\begin{bmatrix} 1 & 3 & 5 & 0 & 10 \\ 0 & 4 & 0 & 2 & 6 \\ 0 & 0 & 0 & 8 & 8 \end{bmatrix}$$

Precise Projection in KS

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- 2 Howellize the matrix
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Example (Project onto x and x')

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$$\begin{array}{ccc} x & x' & 1 \\ \left[\begin{array}{ccc} 0 & 8 & 8 \end{array} \right] \end{array}$$

How can we adapt KS to
directly model w -bit ints?

Use Howell form for projection!

Which is more precise, KS or MOS?

MOS Definition

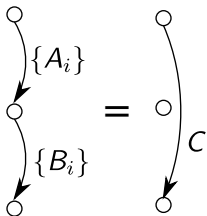
MOS element: a set of matrices of w -bit ints;
every affine combination those matrices
may transform the initial state

Example

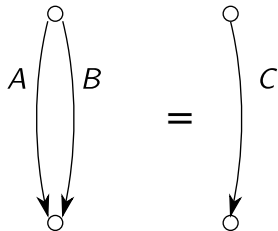
$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \right\} : \quad \exists k: \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

See: Müller-Olm and Seidl, TOPLAS 2007

MOS Compose



$$C = \text{Basis} \{B_j A_i\}$$



$$C = \text{Basis} \{A \cup B\}$$

Basis via Howellize

Can use Howellize for the *Basis* function

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This allows easy equality checking!

Non-Affine Constraints

MOS can represent non-affine constraints!

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Example

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} : \quad \exists k: \begin{bmatrix} k & 1-k & 0 \\ k & 1-k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

Non-Affine Constraints

MOS can represent non-affine constraints!

Example

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\} : \quad \exists k : x' = y' = y + k(x - y)$$

Example

$\llbracket \text{assume}(x = 5) \rrbracket : x = x' \wedge y = y' \wedge x = 5$

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One of the best MOS transformers is $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

Example

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One of the best MOS transformers is $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$

MOS cannot represent assumes!

Which is more precise, KS or MOS?

KS and MOS are incomparable

Which is more efficient, KS or MOS?

Naive Algorithms

For k variables:

	KS	MOS
Element size	$O(k^2)$	$O(k^4)$
Join	$O(k^3)$	$O(k^6)$
Compose	$O(k^3)$	$O(k^7)$

Fast Algorithms

If ring matrix multiplication is $O(k^\alpha)$, then:

	KS	MOS
Element size	$O(k^2)$	$O(k^4)$
Join	$O(k^\alpha)$	$O(k^{2\alpha})$
Compose	$O(k^\alpha)$	$O(k^{4+\alpha})$

Experimental Setup

For eight small programs (500-4000 instructions):

- 1 Compute MOS and KS elements on program edges
- 2 Perform two-phase queries at the beginning of basic blocks that end in branches
- 3 Compare MOS and KS precision at each query point

Experimental Setup

Symbolic abstraction to build KS elements

Operator reinterpretation to build MOS elements

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Symbolic abstraction to build KS elements

Operator reinterpretation to build MOS elements

SMT is used only in devising initial KS elements

Not in KS's analysis phases; nowhere in MOS

Experimental Results: Precision

KS (with symbolic abstraction) was at least as precise as MOS (with operator reinterpretation) at **every** query point

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KS (with symbolic abstraction) was at least as precise as MOS (with operator reinterpretation) at **every** query point

If this holds for KS with operator reinterpretation, then MOS's non-affine constraints don't help for real programs

Experimental Results: Construction Time

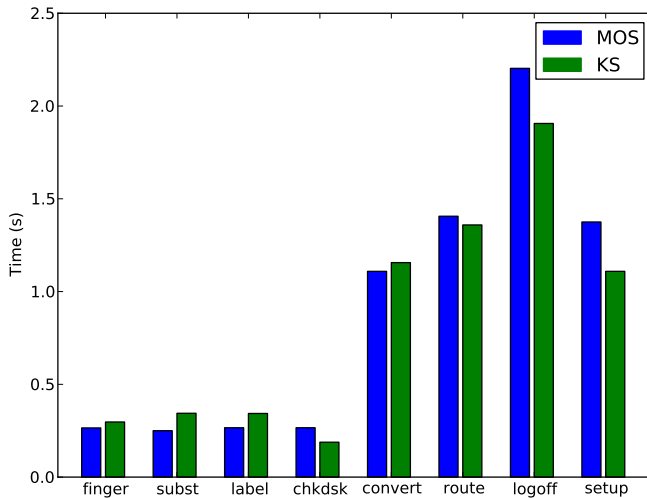
Constructing KS via symbolic abstraction took 325 times longer than constructing MOS via operator reinterpretation

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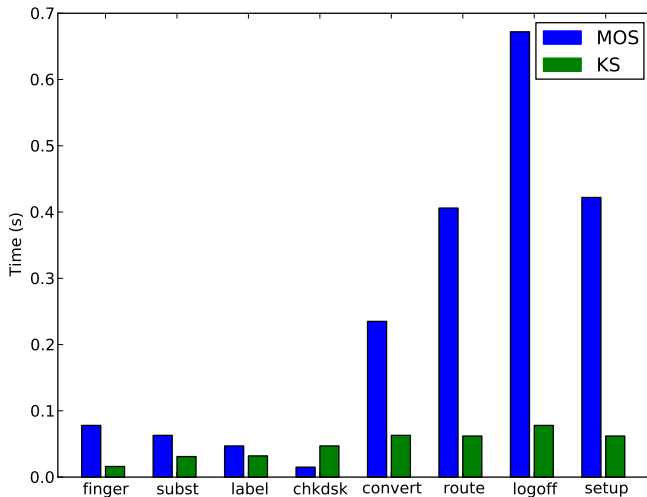
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Constructing KS via operator reinterpretation: coming soon

Experimental Results: Phase 1 Time



Experimental Results: Phase 2 Time



Experimental Conclusions

Overall, KS analysis time was 91% of MOS analysis time

Phase 1 time, KS/MOS: 94%

Phase 2 time, KS/MOS: 20%

Seems that KS analysis is somewhat faster than MOS on real inputs

Technical Highlights

- Howell form allows projection in KS
- Howell form is a normal form for KS and MOS
- MOS can capture non-affine constraints

Conclusions

- KS for w -bit ints:
 - Needs no bit blasting
 - Now applies to larger programs
- KS and MOS are mathematically incomparable
- KS analysis is more efficient than MOS, in theory and (provisionally) in practice